ELECTROMAGNETIC-GRAVITATIONAL CONVERSION CROSS SECTIONS IN EXTERNAL ELECTROMAGNETIC FIELDS

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Abstract

The classical processes: the conversion of photons into gravitons in the static electromagnetic fields are considered by using Feynman perturbation techniques. The differential cross sections are presented for the conversion in the electric field of the flat condesor and the magnetic field of the selenoid. A numerical evaluation shows that the cross sections may have the observable value in the present technical scenario.

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1. **Introduction**

The fundamental consequence of the relativistic theory of gravitation is the existence of gravitational waves(GWs). Einstein was the first to investigate them [1]. For a long time (up to the early sixties) gravitational radiation has been thought of as a phenomennon of great theoretical interest, but of no relevance to the real world. However, the Dyson's remark [2] about the GW " energy flux " from star $F_{gw} \sim 3.629 \times 10^{59}$ erg/sec gave an extra stimulus to consider GWs as real entities. Because of technical restrictions one mainly pays attention to the outer- space sources of GWs such as supernovas, binary stars, etc [3] . However these experiments are partly passive, because they depend on objective conditions.

The study on the interaction between electromagnetic (EM) and gravitational fields is a significant work of research on gravitational radiation. At the present technical level in the laboratory, it has been proved by means of mechanics that it is as yet difficult to generate GWs which are strong enough to be detected by current detectors. Therefore physicists have transfered their interests to the strong EM field and take it as one of the possible sources in the laboratory.

At present, the best hope is to take the interaction between GW and the EM field as a basis for new methods for detecting GWs. These new possible methods would be especially suited to GW with frequencies so high that they cannot be detected by mechanical antenna, which are limited by the properties of the antenna material. The EM detection of the gravitons has been considered by many authors [5]-[9]. In Ref. [6] many properties of the EM field which is generated by a charged condensor in the field of a GW were considered by using purely classical method. The results have a methodical meaning only since the size of the condensor was assumed to be infinite, however, some of them coincide with ours. In Ref. [9] we have considered the conversion of gravitons into photons in the periodic external EM field by using Feynman diagram techniques. Cross sections have been calculated in the quasi-static limit.

Here we lay out the theoretical principles of the most hopeful effect in detection of the gravitons in eathly conditions : it is the generation of gravitons by photons in an external EM field. The main advantages of this effect are following : first, it is the first order of the perturbation theory, second, since the EM field is classical theory we can increase cross sections as much as possible by increasing the intensity of the field or the volume containing the field.

The paper is organized as follows: some notations and general matrix element are given

in section 2. We consider the process in static electric field in section 3 and the process in magnetic field in section 4. Finally in section 5 we present our conclusions.

2. Photoproduction of gravitons

Let us consider the process in which the initial state has the photon γ with momentum q and the external electromagnetic field (EM_{ex}) and the final state has the graviton g with momentum p and the above mentioned electromagnetic field :

$$\gamma + EM_{ex} \longrightarrow g + EM_{ex}. \tag{1}$$

We shall work with the linear approximation and use the quantization of Gupta [10].

From the interaction Lagrangian for the gravitational field with the EM field in the linear approximation, the interaction Lagrangian corresponding to this process has the following form [5]-[6]

$$L_{int}(A, F_{class}, g) = \frac{\kappa}{2} \eta_{\nu\beta} h_{\mu\alpha} F^{\mu\nu} F^{\alpha\beta}_{class}$$

where $h_{\mu\alpha}$ represents gravitational field, $F_{\mu\nu}$ - EM strength tensor, $F^{\alpha\beta}_{class}$ - external EM strength tensor, $\eta_{\nu\beta} = \text{diag}(1, -1, -1, -1)$, $\kappa = \sqrt{16\pi G}$.

Using the Feynman rules we get the following expression for the matrix element [5]-[6]

$$\langle p|M_{ex}|q\rangle = \frac{\kappa}{4(2\pi)^2 q_0} \varepsilon^\lambda(\vec{q}, \sigma) \varepsilon^{\mu\alpha}(\vec{p}, \tau)(\eta_{\lambda\beta} q_\mu - q_\beta \eta_{\mu\lambda}) \int_V e^{i(\vec{q}-\vec{p})\vec{r}} F^\beta_{\alpha class} d\vec{r} \tag{2}$$

where $\varepsilon^\lambda(\vec{q}, \sigma)$ and $\varepsilon^{\mu\alpha}(\vec{p}, \tau)$ represent the polarization tensor of the photon and the graviton, respectively.

Expression (2) is valid for an arbitrary external EM field. In the following we shall use it for two cases, namely the generation in the electric field of a flat condensor and in the static magnetic field of the selenoid. Here we use the following notations: $q \equiv |\vec{q}|, p \equiv |\vec{p}|$ and θ is the angle between $\vec{p}$ and $\vec{q}$. Note that since both photon and graviton are massless, the energy conservation gives us $p = q$.

3. Photoproduction in electric field

Let us consider the generation of gravitons in the homogeneous electric field of a flat condensor of size $a \times b \times c$. We shall use the coordinate system with the x axis parallel to the direction of the field, i.e.,

$$F^{10} = -F^{01} = E.$$

Then the matrix element is given by

$$\langle p|M_e|q\rangle = -\frac{\kappa}{4(2\pi)^2} \varepsilon^i(\vec{q}, \sigma) \varepsilon_{i1}(\vec{p}, \tau) E(\vec{q} - \vec{p}), i = 1, 2, 3 \tag{3}$$

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where

$$E(\vec{q} - \vec{p}) = \int_V e^{i(\vec{q} - \vec{p})\vec{r}} E(\vec{r}) d\vec{r}.$$

For a homogeneous field of intensity E we have

$$E(\vec{q} - \vec{p}) = 8E \frac{\sin[\frac{1}{2}a(q_x - p_x)]\sin[\frac{1}{2}b(q_y - p_y)]\sin[\frac{1}{2}c(q_z - q_z)]}{(q_x - p_x)(q_y - p_y)(q_z - p_z)}. \tag{4}$$

In order to distinguish the processes in an electric field with those in a magnetic field, a subscript e has been supplemented to $M : M_e$.

Squaring the matrix element we meet the expression of summing up over the polarizations of photons and gravitons. Using the following well- known formulas [8]

$$t^{ij}(k) \equiv \sum_\sigma \varepsilon^i(\vec{k}, \sigma)\varepsilon^j(\vec{k}, \sigma) = \delta^i_j - \frac{k^i k^j}{k^2}$$

$$t^{ij,mn}(k) \equiv \sum_\sigma \varepsilon^{ij}(\vec{k}, \sigma)\varepsilon^{mn}(\vec{k}, \sigma) = -t^{ij}(k)t^{mn}(k) + t^{im}(k)t^{jn}(k) + t^{in}(k)t^{jm}(k)$$

$$i, j, m, n = 1, 2, 3.$$

we find easily

$$\sum_{\sigma\tau} \varepsilon^i(\vec{q}, \sigma)\varepsilon^j(\vec{q}, \sigma)\varepsilon_{i1}(\vec{p}, \tau)\varepsilon_{j1}(\vec{p}, \tau) = \left(1 - \frac{p_x^2}{p^2}\right)(1 + \cos^2\theta) \tag{5}$$

From Eq. (3) and Eq. (5) the differential cross section for this process is given

$$\frac{d^e\sigma(\gamma \to g)}{d\Omega} = \frac{\kappa^2}{32(2\pi)^2}|E(\vec{q} - \vec{p})|^2 \left(1 - \frac{p_x^2}{p^2}\right) q^2(1 + \cos^2\theta) \tag{6}$$

Finally substituting Eq. (4) into Eq. (6) we find the differential cross section of the generation of gravitons in the electric field of a flat condensor of size $a \times b \times c$

$$\frac{d^e\sigma(\gamma \to g)}{d\Omega} = \frac{2\kappa^2 E^2}{(2\pi)^2} \left[\frac{\sin(\frac{1}{2}a(q_x - p_x))\sin(\frac{1}{2}b(q_y - p_y))\sin(\frac{1}{2}c(q_z - q_z))}{(q_x - p_x)(q_y - p_y)(q_z - p_z)}\right]^2$$

$$\times q^2(1 + \cos^2\theta)\left(1 - \frac{p_x^2}{p^2}\right). \tag{7}$$

Let us consider the following cases:

a) The momentum of photon is parallel to the z axis, i.e., $q^\mu = (q, 0, 0, q)$. We have then

$$p_x = q\sin\theta\cos\varphi, \ p_y = q\sin\theta\sin\varphi, \ p_z = q\cos\theta. \tag{8}$$

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where φ is the angle between the x axis and the projection of $\vec{p}$ on the xy plane.

Substitution of Eq. (8) into Eq. (7) we get

$$\frac{d^e\sigma(\gamma \to g)}{d\Omega} = \frac{2\kappa^2 E^2}{(2\pi)^2 q^4}\left[\sin\frac{aq\sin\theta\cos\varphi}{2}\sin\frac{bq\sin\theta\sin\varphi}{2}\sin\frac{cq(1-\cos\theta)}{2}\right]^2$$
$$\times[\sin^2\theta\sin\varphi\cos\varphi(1-\cos\theta)]^{-2}(1+\cos^2\theta)(1-\sin^2\theta\cos^2\varphi). \qquad (9)$$

From Eq. (9) we have

$$\frac{d^e\sigma(\gamma \to g)}{d\Omega} = \frac{\kappa^2 E^2}{16(2\pi)^2}V^2 q^2, \qquad V \equiv a.b.c \qquad (10)$$

for $\theta \approx 0$ ($\theta \ll \frac{2}{aq}, \frac{2}{bq}, \frac{2}{\sqrt{cq}}$) and

$$\frac{d^e\sigma(\gamma \to g)}{d\Omega} = \frac{\kappa^2 E^2 a^2}{2(2\pi)^2 q^2}\sin^2\frac{bq}{2}\sin^2\frac{cq}{2} \qquad (11)$$

for $\theta=\frac{\pi}{2}$, $\varphi=\frac{\pi}{2}$ and

$$\frac{d^e\sigma(\gamma \to g)}{d\Omega} = 0 \qquad (12)$$

for $\theta=\frac{\pi}{2}$, $\varphi=0$.

Expressions (10)-(12) show that the probabilities of the gravitons generation are largest in the direction of photon motion. Formula (10) shows that the differential cross section depends quadratically on the square of the intensity of the electric field E, the volume V of the condensor and the photon momentum q. Expression (11) shows that in order to get $\sigma \approx 10^{-30}$cm^2 one has to use an electric field of intensity $E \approx 10^{10}/a\lambda$, while expression (10) shows that in order to get the same cross section we need an electric field of intensity $E \approx 10^9\lambda/V$ only. When $q \to 0$ the right hand side of Eq. (11) is proportional to q^2.

b) The momentum of photon is parallel to the x axis, i.e., $q^\mu = (q, q, 0, 0)$.

In the spherical coordinate system the components of $\vec{p}$ are given as follows

$$p_x = q\cos\theta, \ p_y = q\sin\theta\cos\varphi', \ p_z = q\sin\theta\sin\varphi'. \qquad (13)$$

where φ' is the angle between the y axis and the projection of $\vec{p}$ on the yz plane. Substituting Eq. (13) into Eq. (7) we get

$$\frac{d^e\sigma(\gamma \to g)}{d\Omega'} = \frac{2\kappa^2 E^2}{(2\pi)^2 q^4}\left[\sin\frac{aq(1-\cos\theta)}{2}\sin\frac{bq\sin\theta\cos\varphi'}{2}\sin\frac{cq\sin\theta\sin\varphi'}{2}\right]^2$$
$$\times[\sin^2\theta\sin\varphi'\cos\varphi'(1-\cos\theta)]^{-2}(1-\cos^4\theta). \qquad (14)$$

From Eq. (14) we have

$$\frac{d^e\sigma(\gamma \to g)}{d\Omega'} = 0$$

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for $\theta \approx 0$ ($\theta \ll \frac{2}{aq}, \frac{2}{bq}, \frac{2}{\sqrt{cq}}$) and

$$\frac{d^e\sigma(\gamma \to g)}{d\Omega'} = \frac{\kappa^2 E^2 b^2}{2(2\pi)^2 q^2} \sin^2 \frac{aq}{2} \sin^2 \frac{cq}{2} \tag{15}$$

for $\theta = \frac{\pi}{2}$, $\varphi' = \frac{\pi}{2}$ and

$$\frac{d^e\sigma(\gamma \to g)}{d\Omega'} = \frac{\kappa^2 E^2 c^2}{2(2\pi)^2 q^2} \sin^2 \frac{aq}{2} \sin^2 \frac{bq}{2} \tag{16}$$

for $\theta = \frac{\pi}{2}$, $\varphi' = 0$.

From Eqs. (15) and (16) we see that if $b = c$ the probabilities of the graviton generation are the same in the y- and z- directions, and if $a = b = c$ the r.h.sides of Eqs. (11), (15), and (16) are the same. The differential cross-sections in Eq. (15) and Eq. (16) also depend on quadratically on the square of the intensity the same as in Eqs. (10) and (11).

4. Photoproduction in magnetic field

Assumming that the direction of the magnetic field is parallel to the z axis, i.e.,

$$F^{12} = -F^{21} = B. \tag{17}$$

For the above mentioned process we get the matrix element:

$$\begin{aligned}
\langle p|M_{g\gamma}^m|q\rangle &= \frac{\kappa}{4(2\pi)^2 q_0} B(\vec{p} - \vec{q})\varepsilon^i(\vec{q}, \sigma)[\varepsilon^{j1}(\vec{p}, \tau)(\eta_{i2}q_j - q_2\eta_{ji}) - \\
&\quad \varepsilon^{j2}(\vec{p}, \tau)(\eta_{i1}q_j - \eta_{ji}q_1)]
\end{aligned} \tag{18}$$

where

$$B(\vec{q} - \vec{p}) = \int_V e^{i(\vec{q}-\vec{p})\vec{r}} B d\vec{r}. \tag{19}$$

Now we calculate integral (19). Suppose that the magnetic field is homogeneous in the selenoid with the radius R and the lenght h. In the cylindrical coordinates, it becomes

$$B(\vec{q}-\vec{p}) = B\int_0^R \varrho d\varrho \int_0^{2\pi} \exp\{i[(q_x - p_x)\cos\varphi + (q_y - p_y)\sin\varphi]\} \int_{-h/2}^{h/2} \exp[i(q_z - p_z)z]dz \tag{20}$$

After some manupulations Eq. (20) can be written as follows

$$B(\vec{q} - \vec{p}) = \frac{4\pi BR}{\sqrt{n_x^2 + n_y^2}(p_z - q_z)} J_1\left(R\sqrt{n_x^2 + n_y^2}\right) \sin\left(\frac{h(p_z - q_z)}{2}\right). \tag{21}$$

where $n_x \equiv p_x - q_x, n_y \equiv p_y - q_y$ and J_1 is the one-order spherical Bessel function [11]. Substituting (21) into (18) we find after cumbersome calculations

$$\frac{d\sigma^m(\gamma \to g)}{d\Omega} = \frac{\kappa^2 R^2 B^2 \sin^2\left[\frac{(q_z - p_z)h}{2}\right] J_1^2\left(R\sqrt{(q_x - p_x)^2 + (q_y - p_y)^2}\right)}{16(q_z - p_z)^2 \left[(q_x - p_x)^2 + (q_y - p_y)^2\right]}$$

$$\times \quad \left((2q^2 - p_x^2 - p_y^2)\sin^2\theta + 4(p_x q_x + p_y q_y)\cos\theta - \frac{4}{q^2}q_x p_x q_y p_y\right.$$

$$- \quad \left.2(q_x^2 + q_y^2) + \frac{2}{q^2}(q_x^2 p_y^2 + q_y^2 p_x^2)\right). \tag{22}$$

When the momentum of the graviton is parallel to the z axis (the direction of the magnetic field), the differential cross section vanishes for $\theta = 0$ and

$$\frac{d\sigma^m(\gamma \to g)}{d\Omega} = \frac{\kappa^2 R^2 B^2}{16q^2} \sin^2\left(\frac{qh}{2}\right) J_1^2(Rq) \tag{23}$$

for $\theta = \frac{\pi}{2}$.

Now we consider the case in which the momentum of the graviton is parallel to the x axis, i.e., $q^\mu = (q, q, 0, 0)$. Substituting (13) into (22) and note that

$$\lim_{p \to q} \frac{J_1\left(R(p - q)\right)}{p - q} = \frac{R}{2}$$

we find

$$\frac{d\sigma^m(\gamma \to g)}{d\Omega'} = \frac{\kappa^2 V^2 B^2 p^2}{128\pi^2} \tag{24}$$

for $\theta = 0$ and

$$\frac{d\sigma^m(\gamma \to g)}{d\Omega'} = \frac{\kappa^2 h^2 R^2 B^2}{128} J_1^2(Rp\sqrt{2}) \tag{25}$$

for $\theta = \frac{\pi}{2}$, $\varphi' = 0$ and

$$\frac{d\sigma^m(\gamma \to g)}{d\Omega'} = 0 \tag{26}$$

for $\theta = \frac{\pi}{2}$, $\varphi' = \frac{\pi}{2}$.

From Eq. (24) we see that the differential cross section in the direction of graviton motion depends quadratically on the magnitude B, the volume V of the selenoid and the graviton momentum p.

From Eq. (25) it follows that the differential cross section vanishes when $p_n = \frac{\mu_n}{R\sqrt{2}}$ with $n = 0, \pm1 \pm 2...$and has its largest value

$$\frac{d\sigma^m(g \to \gamma)}{d\Omega'} \approx 3.2 \times 10^{-50} h^2 R^2 B^2 J_1^2(\mu_n') \tag{27}$$

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for $p_n = \frac{\mu'_n}{R\sqrt{2}}$. Where μ_n and μ'_n are the roots of $J_1(\mu_n) = 0$ and $J'_1(\mu'_n) = 0$.

5. **Discussion**

The following consequences may be obtained from our results

1 - It is the best for experiments when the momentum of photons is perpendicular to the EM field and in this case, the conversion cross sections are largest in the direction of the photon motion.

2 - In C.G.S units, formulas (10) and (11) have the forms, respectively

$$\frac{d\sigma_\parallel(\gamma \to g)}{d\Omega} \approx 1.32.10^{-49} \frac{V^2 E^2}{\lambda^2} \tag{28}$$

$$\frac{d\sigma_\perp(\gamma \to g)}{d\Omega} \approx 1.32.10^{-51} a^2 E^2 \lambda^2 \sin^2 \frac{b\pi}{\lambda} \sin^2 \frac{c\pi}{\lambda} \tag{29}$$

Note that when $\lambda \gg b, c$ (28) becomes

$$\frac{d\sigma_\perp(\gamma \to g)}{d\Omega} \approx 1.32.10^{-49} \frac{V^2 E^2}{\lambda^2}$$

Suppose that the size of the condensor is, $1m \times 1m \times 1m$, the intensity of the electric field $E = 100 \frac{kV}{m}$ and the photon length $\lambda = 10^{-5}$ cm, the cross section given by (28) is $\frac{d\sigma_\parallel(\gamma \to g)}{d\Omega} \approx 10^{-16} \text{cm}^2$, while $\frac{d\sigma_\perp(\gamma \to g)}{d\Omega} \approx 10^{-46} \text{cm}^2$. The situation is analogous in the case of the magnetic field.

In C. G. S. units Eq. (24) becomes

$$\frac{d\sigma(g \to \gamma)}{d\Omega'} \approx 1.3 \times 10^{-49} \frac{V^2 B^2}{\lambda^2} \tag{30}$$

where λ is the wavelength of graviton and the cross section gives

$$\frac{d\sigma(g \to \gamma)}{d\Omega'} \approx 1.3 \times 10^{-15} \text{cm}^2 \tag{31}$$

for $V = 10^6 \text{cm}^3$, $B = 10^6 \text{cm}^{-1/2} \text{g}^{1/2} \text{s}^{-1}$, $\lambda = 10^{-5}$ cm

3 - Note that only for $\lambda \geq \sqrt{\frac{10V}{a}}$ the cross section in the two directions, namely $\theta \sim 0$ and $\theta = \frac{\pi}{2}$, $\varphi = \frac{\pi}{2}$ have the same order and for all other λ the expression (24) has always advantages.

Finally, note that in Ref. [5] using Feynman perturbation tehniques authors have analyzed the conversion of GWs into EM waves in an uniform electrostatic and magnetic fields in which their background are confirmed to the region between the planes $z = -l/2$ and $z = l/2$. Authors have considered the only case in which both momenta of graviton and photon are parallel to the z axis (this case corresponds with $\theta = 0$ in our results). Therefore,

we see that our results are more realistic. Finally we note again that the cross sections of EM- gravitational conversion may have the observable value in the present technical scenario.

In this work we considered only a theoretical basis for experiments. Other problems with detection will be investigated in the future.

Acknowledgements

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